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THE STRENGTH OF THIN CYLINDRICAL SHELLS AS COLUMNS

ENGINEERING
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A REPORT OF AN INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION

UNIVERSITY OF ILLINOIS

IN COÖPERATION WITH

THE CHICAGO BRIDGE AND IRON WORKS

BY

WILBUR M. WILSON

AND

NATHAN M. NEWMARK

BULLETIN No. 255

ENGINEERING EXPERIMENT STATION

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THE STRENGTH OF THIN CYLINDRICAL SHELLS AS COLUMNS

I. Introduction

1. *Introduction.*—A thin cylindrical shell subjected to compression in the direction of its longitudinal axis may fail either by instability of the shell as a whole, involving bending of the axis, or by local instability of the wall of the shell which may not at all involve lateral distortion of the axis. The former type of failure is that investigated by Euler, and the strength depends on the ratio of length to radius of gyration of the shell. The latter type of failure has been called by various writers secondary flexure, crinkling, or wrinkling, and the wrinkling strength depends on the ratio of thickness to radius of the shell wall. Failure of this type is by formation of characteristic wrinkles or bulges, circular or lobed in shape depending on the properties of the specimen. It is, of course, possible for a combination of both types of failure to occur in a moderately long slender tube.

The strength of a thin cylindrical shell as a column has assumed additional interest among structural engineers because of the recent development of elevated storage tanks having radial-cone bottoms* for which the standpipe functions as a column to carry part of the weight of the tank and its contents. For example, a 1 500 000-gallon tank at Indianapolis, Indiana, has a standpipe 10 ft. 0 in. in diameter, 70 ft. high, and ½ in. in thickness, which serves as a column and which was designed for an axial compressive stress of 5 010 lb. per sq. in.

2. *Object and Scope of Investigation.*—The object of the investigation was to study the type of failure characterized by local wrinkling and to investigate the dependence of the wrinkling strength upon variables of two kinds: (1) physical properties of the material, and (2) geometrical properties of the cylinder, such as length, thickness, radius, and end conditions.

The effect of these variables has been studied experimentally, and an attempt has been made to correlate the results with the theory as presented by Southwell and others. The particular variables studied were:

(1) Thickness–radius ratio in both machined and fabricated specimens.

*Patented by the Chicago Bridge and Iron Works.

(2) Transverse joints in both machined and fabricated specimens.

(3) Length of both machined and fabricated specimens.

(4) Eccentricity of load on short and medium length machined specimens.

A method is proposed for determining the wrinkling strength of shells from the stress–strain curve for the material.

3. *Acknowledgments.*—This investigation is a part of the work of the Engineering Experiment Station of the University of Illinois, of which DEAN M. S. KETCHUM is the director, and the Department of Civil Engineering, of which Prof. W. C. HUNTINGTON is the head. The work was financed by the Chicago Bridge and Iron Works, who furnished the specimens and provided funds from which to pay the direct expenses.

The work was done in consultation with an Advisory Committee of the Chicago Bridge and Iron Works, the members of which are: H. C. BOARDMAN, Chairman, GEORGE T. HORTON, O. A. BAILEY, E. E. MICHAELS, and RALPH GREEN.

II. ELASTIC WRINKLING

4. *General Theory.*—Analytical solutions of the problem of wrinkling or buckling of thin cylindrical shells under axial load are given by Southwell,* Dean,† and others.‡

In these derivations it is assumed that the elastic limit of the material is not exceeded. In general, although different methods of approach are used, the same results are obtained, namely, for a uniform circular bulge or wrinkle,

$$S = \frac{E}{\sqrt{3}} \cdot \frac{1}{\sqrt{1 - \mu^2}} \cdot \frac{t}{R} \qquad (1)$$

where S = unit stress which causes a condition of instability likely to result in wrinkling, for brevity, the critical wrinkling stress.

E = modulus of elasticity of the material.

μ = Poisson's ratio.

t = thickness of the shell wall.

R = mean radius of the shell; that is, the inside radius plus one-half the thickness.

*R. V. Southwell, Phil. Trans., Royal Society, London, Series A, Vol. 213, 1914, p. 187.
†Dean, Proc. Royal Society, London, Series A, Vol. 107, 1925, p. 734.
‡Prescott, Applied Elasticity, p. 530.
R. Lorenz, Zeitschrift des Vereins Deutscher Ingenieure, Vol. 52, 1908, p. 1706.
R. Lorenz, Physikalische Zeitschrift, 1911, p. 241.
S. Timoshenko, Zeitschrift fur Math. u. Phys., Vol. 58, 1910, p. 337.

Southwell, Dean, and Prescott give for a lobed form of wrinkle the equation:

$$S = \frac{E}{\sqrt{3}} \cdot \frac{1}{\sqrt{1-\mu^2}} \cdot \frac{t}{R} \cdot \frac{N^2-1}{N^2+1} \tag{2}$$

where N = number of lobes in the wrinkle.

For a long slender tube, failure does not occur by wrinkling but by the type of buckling investigated by Euler. The unit stress at which buckling is likely to occur in this case is given by the formula,

$$s = c\frac{\pi^2 E}{\left(\frac{L}{r}\right)^2} \tag{3}$$

where s = unit stress at failure.

r = radius of gyration of the cross-section of the shell.

c = a constant depending on the end conditions.

For a very thin shell, $r^2 = \frac{R^2}{2}$, and Equation (3) becomes,

$$s = c\frac{\pi^2 E}{2\left(\frac{L}{R}\right)^2} \tag{4}$$

It may be noted that Equations (1) and (2) for the wrinkling type of failure do not involve the length of the shell. That is, the critical wrinkling stress is independent of the length. Although the unit critical wrinkling stress is independent of the length of the shell, nevertheless the total load-carrying capacity is affected by the ratio of the length to the radius of gyration in the case of long slender shells. For if there is a tendency to buckle the stress will no longer be uniform over a section and failure will occur when the maximum stress on the section becomes equal to the critical wrinkling stress.

Robertson* disagrees with Equation (2), claiming that it should be the same as Equation (1). He points out that in deriving Equation (2) both Southwell and Dean assumed that the axial wave length of the deformations of the shell wall is large. However, experiments have shown that this wave length is small, and with this assumption Southwell's analysis gives the same result for a lobed wrinkle as for a circular wrinkle, upholding Robertson's contention.

*Robertson, Technical Report of the Aeronautical Research Committee, 1928-29, Vol. II, p. 935.

In any case, where the number of lobes is greater than 3, Equation (2) gives substantially the same critical stress as Equation (1).

An expression may be derived for the maximum number of lobes, N, in a lobed form of wrinkle. According to Robertson,*

$$N < \frac{1}{2\sqrt[4]{A}} \tag{5}$$

where

$$A = \frac{1}{12}\left(\frac{t}{R}\right)^2 \frac{1}{1-\mu^2} \tag{6}$$

For a value of μ of 0.3, the expression reduces to

$$N < \frac{0.9091}{\sqrt{\frac{t}{R}}} \tag{7}$$

According to the derivation, the number of lobes may range from zero to the right-hand term of Equation (7). It may be noted that a two-lobed wrinkle is elliptical, a three-lobed wrinkle triangular, etc.

5. *Limitations of Theory.*—Equation (1) of Section 4 is limited in its application to perfect specimens; that is, those in which the form of the cylinder is perfect, and the material is homogeneous and has a constant ratio of strain to stress until failure occurs by wrinkling. In other words, for steel specimens it is limited to values of t/R which are small enough for the theoretical critical wrinkling stress to be less than the proportional limit of the material. For example, for steel having a proportional limit of 30 000 lb. per sq. in., modulus of elasticity of 30 000 000 lb. per sq. in., and Poisson's ratio of 0.3, the value of t/R for *elastic* instability must be less than 0.00166. For a shell four inches in diameter this would correspond to a wall thickness of 0.00332 inches.

It appears, then, that in cylindrical shells or tubes of the proportions ordinarily used, failure could not occur by *elastic* wrinkling because the proportional limit of the material is less than the theoretical wrinkling stress. It is desirable, therefore, to find an expression for the stress causing collapse of a tube of such proportions that a wrinkling failure occurs because of *plastic* instability at a stress equal to or greater than the proportional limit of the material. A method

*Technical Report of the Aeronautical Research Committee, 1928-29, Vol. II, p. 930. This reference contains a typographical error. The correct expression is given in this text.

proposed by Geckeler is given in Section 6, and one proposed by the authors in Section 7.

Southwell's theoretical analysis was developed for specimens of uniform thickness concentrically loaded. Assuming, for eccentric loading, that wrinkling occurs when the unit stress at any point (the sum of the stresses due to flexure and to axial loading) reaches the critical stress (an assumption that is borne out by the results of the experiments herein reported), an expression may be derived for the wrinkling strength of a tube loaded through its geometrical center, but having a known variation in thickness around any cross-section. This variation in thickness has the same effect as eccentricity of load and may appreciably increase the stress at the thinnest point.

The effect of a variation in thickness is determined on the assumption that the inner and outer boundaries of the shell are circular but not concentric. The further assumptions are made that the mean surface, or the surface midway between the inner and outer surfaces, is approximately circular; and that the thickness, consequently the variation in thickness, is small compared to the radius of the tube.

The eccentricity is given by the expression

$$\frac{e}{R} = \frac{a}{2t} \tag{8}$$

and the increase in stress due to the eccentricity resulting from the assumed variation in thickness is given by the expression

$$f_1 = \frac{a}{t} \frac{1 + \frac{1}{2}\frac{a}{t}}{1 - \frac{1}{2}\frac{a^2}{t^2}} f \tag{8a}$$

where R = mean radius of the shell wall.

e = eccentricity of load, equal to distance from center of gravity to geometrical center of mean surface.

t_1 = minimum thickness.

t_2 = maximum thickness, diametrically opposite.

t = average thickness; $t = \frac{t_2 + t_1}{2}$.

a = one-half the maximum variation in thickness; $a = \frac{t_2 - t_1}{2}$.

f = stress for concentric load on uniform section having same area as given section.

f_1 = increase in stress due to variation in thickness.

For small variations, f_1 is approximately equal to $\frac{a}{t} \cdot f$, or the per cent increase in stress is approximately equal to the per cent deviation in thickness from the average.

III. Plastic Wrinkling

6. *Method Presented by Geckeler.*—J. W. Geckeler* has treated the case of plastic wrinkling mathematically, making the assumption that each longitudinal strip of the shell wall acts as a column subjected to lateral forces from the other strips, as well as a compressive load. He considers only the possibility of a uniform circular bulge and gives the equation for the critical stress:

$$S = \frac{\sqrt{K E}}{\sqrt{3\,(1 - \mu^2)}} \cdot \frac{t}{R} \tag{9}$$

in which K is defined by the equation,

$$K = \frac{4\,E_1\,E_2}{(\sqrt{E_1} + \sqrt{E_2})^2}$$

where E_1 = slope of stress–strain curve for any given compressive stress, that is, for the existing stress, S.

E_2 = slope of stress–strain curve for a decreasing compressive stress, approximately equal to E, the initial modulus of elasticity in tension, for structural steel.

The equation has to be solved by trial for any particular case, since E_1 is a function of S.

The constant K is the same as that given by Von Karman† and Nadai‡ for the Eulerian plastic buckling of rectangular columns for

*"Plastisches Knicken der Wandung von Hohlzylindern," Zeitschrift für Angewandte Mathematik und Mechanic, Oct. 1928, V. 8, p. 341.

†"Untersuchungen über Knickfestigkeit," Mitteilungen über Forshungsarbeiten auf dem Gebiete des Ingenierwesens, Vol. 81, 1910, p. 20.

‡"Plasticity," A. Nadai, p. 178.

use in the ordinary formula instead of E. If E_2 is taken equal approximately to E, Equation (9) becomes

$$S = \frac{E}{\sqrt{3}} \cdot \frac{1}{\sqrt{1-\mu^2}} \cdot \frac{t}{R} \cdot \frac{2}{1+\sqrt{\frac{E}{E_1}}} \tag{9a}$$

It is readily seen that Equation (9a) agrees with Equation (1) when $E_1 = E$, or in the region of elastic wrinkling. When E_1 approaches zero, which it does at a yield point, the value of S approaches zero. This would indicate that it is impossible to obtain a critical stress higher than the yield point, no matter what the thickness–radius ratio may be.

7. *Proposed Critical Strain Method.*—A rational method for the treatment of plastic wrinkling is proposed. This method lends itself readily to the use of an empirical constant to correlate the results of experiment with theory.

The theoretical equation for elastic wrinkling, derived by Southwell and others, gives for the critical wrinkling stress

$$S = \frac{E}{\sqrt{3}} \cdot \frac{1}{\sqrt{1-\mu^2}} \cdot \frac{t}{R}$$

The strain at this critical stress is

$$\epsilon = \frac{S}{E} = \frac{1}{\sqrt{3\,(1-\mu^2)}} \cdot \frac{t}{R} \tag{10}$$

since perfect proportionality between stress and strain is assumed in the derivation of Equation (1).

It is evident, then, that for elastic wrinkling the critical strain is independent of the elastic modulus of the material, and depends only on the dimensions of the tube, t and R, and to a very slight extent on Poisson's ratio. The effect of a variation in Poisson's ratio from 0.2 to 0.4 in either Equation (10) or Equation (1) is not appreciable. Consequently, the value of $\mu = 0.3$ has been used in the computations throughout. For this value, Equation (10) becomes

$$\epsilon = 0.60 \frac{t}{R}. \tag{11}$$

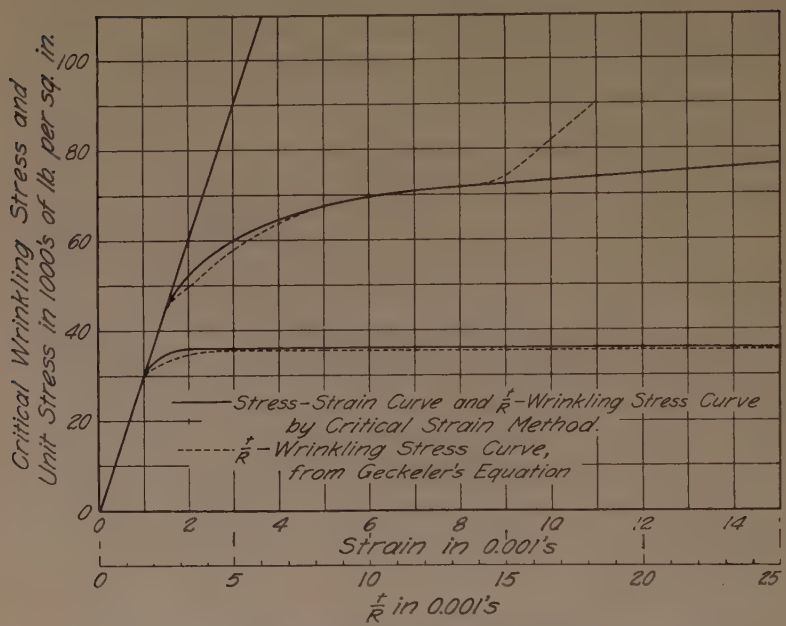
Critical Wrinkling Stress and Unit Stress in 1000's of lb. per sq. in.
100
80
60
40
20
0
Stress-Strain Curve and $\frac{t}{R}$-Wrinkling Stress Curve by Critical Strain Method.
$\frac{t}{R}$-Wrinkling Stress Curve, from Geckeler's Equation
0 2 4 6 8 10 12 14
Strain in 0.001's
0 5 10 15 20 25
$\frac{t}{R}$ in 0.001's

FIG. 1. ASSUMED STRESS-STRAIN CURVES AND CORRESPONDING t/R–WRINKLING–STRESS CURVES

If it is assumed that a tube of given dimensions will fail when the strain reaches the critical wrinkling strain of Equation (10), then this equation is applicable to either *elastic* or *plastic* wrinkling. That is to say, the stress–strain curve for the material in compression is, to the scale indicated by Equation (11), also a wrinkling-strength–t/R curve.

Three assumed typical stress–strain curves for steel are given in Fig. 1. The scale for the wrinkling-strength–t/R curve is also indicated in the figure. In addition, the wrinkling-strength–t/R curves given by Geckeler's equation for the same assumed stress–strain curves are shown plotted on the same coordinates. The difference between the two sets of curves for the wrinkling stress is negligible for the stress–strain curves assumed.

It may be noted that if failure is *elastic*, the total load carried by a tube varies directly as the square of the thickness and is independent of the radius, while if failure is *plastic*, the total load varies directly (approximately) with the area of the cross-section of the tube.

8. *Review of Previous Experimental Investigations.*—Lilly* was one

*Institution of Civil Engineers, Ireland, March 7, 1906, p. 27, and Proc. Inst. Mech. Eng., London, 1905, Part 3 and 4, p. 697.

of the first experimental investigators of this problem. He tested a large number of mild steel tubes varying from $\frac{3}{8}$-in. diameter and 22-gauge thickness to 1-in. diameter and 18 gauge, and found that when the ratio of the length to the radius of gyration was less than 80 for the thinnest tubes, failure occurred by wrinkling. He concluded that "when the failure takes place by secondary flexure (wrinkling), for a large range of the length there is little variation in the breaking load." Lilly reported no control tests, but stated that the ultimate strength in tension of the tubing varied from 62 000 to 85 000 lb. per sq. in.

Mason,* in experiments on mild steel under combined stress made compression tests on hollow cylindrical tubes $2\frac{3}{4}$-in. diameter and 10 gauge, and 3-in. diameter and 14 guage, t/R being 0.089 and 0.052, respectively. He found failure to occur at about the yield point stress, and that, over a small range, length had no effect on the stress required to cause failure by wrinkling.

Popplewell and Carrington† made tests on high tensile strength steel tubes, both hardened and annealed. Unfortunately, they reported no adequate control tests, and defined the "wrinkling stress" as the stress at which the stress–strain relationship for the tube departs appreciably from a straight line. They do not report the ultimate loads. They found that for values of t/R above 0.1, their "wrinkling stress" is the same as the proportional limit of the material in tension, and below this value of t/R, the "wrinkling stress" is directly proportional to t/R.

Robertson‡ concludes, from his very extensive series of experiments, that:

(a) "For tubular struts of mild steel having a yield in compression of 22 tons per sq. in., the strength depends on the yield stress and not on the wrinkling stress provided t/R is greater than 0.022.

(b) "For short specimens of thin tubes, tested in compression, failure will occur either at the yield or at a stress (below the yield point) which is some fraction of the Southwell value (Equation 1)."

The value of this fraction, from Robertson's experiment, varies from 0.4 for tubes of drawn mild steel and nickel-chrome steel to 0.6 for tubes of the high strength steel used in aircraft construction.

Robertson seems to be the only one who has conducted tests on tubes of such thickness that elastic failure, or failure at a stress materially below the yield point, takes place.

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*Mason, Proc. Inst. of Mech. Eng., Parts 3 and 4, 1909, p. 1205.
†Popplewell and Carrington, Proc. Inst. Civil Engineering, Vol. 203, 1916-17, p. 381.
‡Robertson, Technical Report of the Aeronautical Research Committee, 1928-29, Vol. II, p. 935.

IV. Description of Tests and Apparatus

9. *Machined Specimens.*—Tests were made on machined specimens 4 in. and 14 in. in diameter and varying in length from 1¾ in. to 20 in., made of lap-welded steel pipe, wrought iron pipe, Shelby seamless steel tubing, and hot-rolled seamless steel tubing.

Nearly all the specimens had ½-in. collars at the ends, which acted as stiffeners, but which were primarily for the purpose of allowing apparatus to be attached and strain gage holes to be drilled without injuring the thin section of the tube. The material and nominal dimensions of the different groups of specimens are given in Table 1.

The specimens made of Shelby seamless tubing were machined to the desired thickness by turning the outside only; all other specimens listed in Table 1 were machined both inside and outside to the desired thickness.

Thickness measurements of the machined specimens were made with an instrument on the principle of a micrometer caliper with a deep throat. The tube to be measured was supported on three round-ended pins embedded in a 2-in. by 0.25-in. steel bar in such a manner that the tube always rested on the center pin and one of the end pins. A steel arm was attached to the bar and supported an Ames dial micrometer graduated to thousandths of an inch, placed so that the plunger moved vertically over the center pin. A zero reading was taken with the plunger in contact with the pin and the tube supported beneath the bar so that its weight was carried in the same manner as if it were being supported on the pins for a measurement. In general the thickness was measured at eight points around the circumference on circumferential gage lines from 0.5 in. to 2.5 in. apart, depending on the length of the specimen.

Stress–strain diagrams for the specimens of series 1 and 5 were obtained during the test by means of an autographic apparatus. The load on the specimen was indicated by the vertical movement of a pencil on the record. This vertical movement was controlled by the hand wheel operating the poise on the testing machine. The deformation of the specimen, greatly multiplied, was indicated by a rotation of the record drum.

Deformations during the test of the specimens of series 6 were measured by means of an apparatus of the type commonly used for determining the strain in concrete cylinders in compression. This apparatus gave the average deformation of opposite sides of the specimen.

TABLE 1
DESCRIPTION OF MACHINED SPECIMENS

Series	Material	Nominal Dimensions			Remarks
		Diameter in.	Length in.	Thickness in.	
A	Lap-welded steel pipe	4	2 to 8	0.02, 0.04, 0.06	Collars at ends.
B	Wrought-iron pipe	4	8	0.04	Offset at center, collars at ends.
D	Shelby seamless steel tubing	4	1.75 to 20	0.02	No collars.
1	Shelby seamless steel tubing	4	8	0.015, 0.03	Collars at ends; from same piece of tubing as specimens of Series D.
2	Shelby seamless steel tubing	4	4, 16	0.02	Collars at ends.
5	Shelby seamless steel tubing	4	2 to 20	0.015	Collars at ends.
6	Hot-rolled seamless steel tubing	14	10	0.01 to 0.03	Collars at ends.

Deformations of all other specimens were obtained with a strain gage, the gage holes being drilled in the collars of the specimen, except in the case of series D, for which the gage holes were drilled in the bearing blocks used in the test. This latter method gave values for the strain which are believed to be erroneous.

All the 4-in. specimens were tested in a 30 000-lb. Riehle testing machine. A special bearing block or plug in contact with the end of the specimen had a cylindrical projection that fitted snugly on the inside and extended into the end of the specimen. Generally a spherical bearing block was used in conjunction with the plug, and was wedged into place after a small initial load had been applied.

All the 14-in. specimens were tested in a 200 000-lb. Riehle testing machine. The bottom of the specimen rested directly on a fixed bearing block, and the inside at the bottom was stiffened by a ring that fitted snugly into the specimen. The upper bearing block was a combination plug and block of the same general type as that used for the small machined specimens and described in the previous paragraph.

For the specimens of series 2, the eccentric loading was accomplished by means of a $\frac{5}{16}$-in. steel ball placed in a shallow conical hole in the top bearing plug. The bottom bearing plug rested on a roller nest, the direction of motion of the rollers being in the direction of eccentricity of load.

During the tests of specimens of series A and B measurements were taken of the external diameter at various places. However, the change in diameter was evidently less than the tolerance of the instrument, 0.0005 in., and the measurements were not made in subsequent tests.

Coupons for tension control tests were cut from the same pieces of pipe as the specimens of series A, B, and 6. These coupons were flat, had approximately the same thickness as the finished specimens they were to represent, and had the same general proportions that are usually used for control specimens of thin plates. The 4-in. Shelby tubing, however, was too thin to permit the machining of coupons of this type from it. Hence a tubular tension control specimen was used and fittings devised for testing it. The tension specimens were machined to the same thickness as the shells which they represent. Strains were read with a microscope strain gage using targets painted on the specimen.

10. *Fabricated Specimens.*—Tests were made on three groups of fabricated specimens, designated as series E, 7, and 8. There were

FIG. 2. TRUNCATED–CONE BEARING BLOCK AS USED TO LOAD SHELL 80 INCHES IN DIAMETER

four specimens in series E, 6 ft. high, 6 ft. 8 in. in diameter, and 0.25 in. thick. Two of the cylinders had a welded butt girth seam, and two a riveted lap girth seam. The vertical seams in each cylinder were of the same type as the girth seams. The cylinders had 3-in. by 3-in. by $\frac{5}{16}$-in. angles spot-welded to the shell around top and bottom, but contact with the loading head and bed of the testing machine was made by the shell alone, the backs of the angles being placed about $\frac{1}{8}$ in. away from the ends of the plates.

The twelve specimens of series 7 were made by welding $\frac{1}{32}$-in. hot-rolled steel plate. All the specimens were 30 in. high; the diameters were 10, 20, 30, 40, 50, and 60 in. The 10-, 20-, and 30-in. specimens were made of one plate each and had one vertical seam. The larger specimens had two vertical seams. The ends of all except the 10- and 20-in. specimens were held cylindrical in shape during

the tests by means of a disk, cut from 1-in. by 8-in. redwood plank, that fitted into the end of the specimen.

The six specimens of series 8 were fabricated by butt-welding ⅛-in., medium-grade, hot-rolled steel plate. The 6-ft. specimens were made from one plate and had only one vertical seam. The longer specimens were made up of 5-ft. sections so connected that the vertical seams of adjacent sections were diametrically opposite. All the specimens were 34 in. in diameter.

All the fabricated specimens, except the 10-in. specimens of series 7, were tested in a 3 000 000-lb. Southwark-Emery hydraulic testing machine. The specimens of diameter greater than 40 in. were loaded by means of the truncated-cone top bearing block shown in Fig. 2. For series 7 and 8 shims were used between the ends of the shell and the bearings to fill gaps caused by unevenness of the specimen or bearing. For series E, measurements were taken at various loads of the width of gap, but the gaps were not filled.

For series E and 8 the strain was measured on eight 8-in. vertical gage lines spaced at equal distances around the circumference, and profile measurements were taken from four wires spaced 90 degrees apart and stretched from top to bottom of the cylinder. No measurements were taken during the tests of the specimens of series 7.

Thickness measurements of all the fabricated specimens were made only at the ends, at eight points around the circumference. Tension control coupons were cut from the parent plates for series 7, and from the cylinders after test for the other series.

V. Results of Tests

11. *General.*—The wrinkling stresses for all the centrically loaded machined specimens are given in Tables 2 and 3, and for the fabricated specimens in Table 4. Stress–strain curves for some of the 4-in. Shelby tubes of series D are given in Fig. 3. Stress–strain diagrams for tension and compression are compared in Fig. 4. These diagrams were plotted from strains measured with a strain gage. Diagrams reproduced from the records of an autographic extensometer are shown in Fig. 5. The diagrams of Fig. 6 show the relation between the stress and strain for the 14-in. specimens. It may be seen from these curves that, in general, wrinkling was accompanied by a considerable sudden drop in load. The curves for series 6 in particular show that release of load is accompanied by some recovery of strain. The set upon full release of load is due primarily to the

TABLE 2

RESULTS OF TESTS, MACHINED SPECIMENS

Specimen No.	Series	Internal Diameter in.	Length		$\frac{t}{R}$ Ratio	Maximum Average Stress lb. per sq. in.	Corrected† Stress lb. per sq. in.	Ratio to Theoretical Wrinkling Stress	Number of Lobes	Theoretical Maximum Number of Lobes
			End to End in.	Between Shoulders in.						
1	A	3.625	8.49	7.47	0.0122	31 400	31 400	0.144	6	8
2		3.635	8.45	7.45	0.0204	38 100	38 100	0.105	3	6
3		3.653	8.01	7.00	0.0349	38 400	38 400	0.060	2	4
5		3.626	8.49	7.51	0.0129	39 950	39 950	0.174	5	7
6		3.642	8.50	7.50	0.0235	36 800	36 800	0.088	3	5
7		3.631	7.99	6.97	0.0353	40 700	40 700	0.065	0	4
15		3.652	4.51	3.50	0.0130	40 000	40 000	0.172	4	7
16		3.652	2.52	1.52	0.0126	40 600	40 600	0.180	0	8
D1	D	3.871	1.75	1.75	0.01018	49 400	49 400	0.267	0	8
D2		3.871	3.75	3.75	0.01022	53 500	53 500	0.289	6	8
D3		3.871	5.75	5.75	0.01018	46 400	46 400	0.246	0	8
D4		3.871	7.75	7.75	0.01104	48 000	48 000	0.240	irreg.	8
D5		3.871	19.87	19.87	0.00997	50 600	50 600	0.280	0	9
110	1	3.871	8.49	7.50	0.01503	52 800	55 500	0.204	5	7
111		3.871	8.50	7.54	0.00752	34 650	40 400	0.297	8	10
112		3.871	8.49	7.54	0.01482	51 000	52 600	0.196	5	7
113		3.871	8.50	7.53	0.00736	42 500	43 700	0.328	8	10
510	5	3.873	2.50	1.53	0.00684	46 800	55 200	0.445	irreg.	11
511		3.873	6.49	5.55	0.00740	41 600	45 500	0.339	7	10
512		3.873	10.50	9.53	0.00731	32 700	36 000	0.272	7	10
513		3.873	19.98	19.05	0.00705	52 400	54 000	0.422	8	10
514		3.873	2.50	1.55	0.00694	47 300	51 300	0.408	8	11
515		3.873	6.49	5.56	0.00710	46 200	52 600	0.408	7	10
516		3.873	10.49	9.54	0.00720	36 400	50 600	0.388	7	10
517		3.873	20.00	19.07	0.00720	27 420	39 600	0.303	irreg.	10
610	6	13.64	10.49	9.50	0.00209	14 140	14 700	0.415	10	19
611		13.64	10.49	9.58	0.00485	31 400	31 400	0.382	8	13
612		13.64	10.52	9.50	0.00303	22 650	23 000	0.448	10	16
613		13.64	10.50	9.49	0.00230	21 450	22 100	0.569	12	18
614		13.64	10.50	9.53	0.00131	7 710	8 010	0.361	12	25
615		13.64	9.66*	9.18	0.00196	16 510	16 590	0.500	12	20
616		13.64	10.46	9.46	0.00294	21 620	21 800	0.437	10	16
617		13.64	10.50	9.48	0.00425	30 120	30 300	0.422	11	13

*Collar at one end only.

†Average stress corrected for variations in thickness of shell.

TABLE 3
RESULTS OF TESTS, MACHINED SPECIMENS WITH OFFSET SHELLS

Specimen No.	Series	External Diameter at Center in.	Length		Eccentricity of Offset in.	$\frac{t}{R}$		Maximum Average Stress		Ratio to Theoretical Wrinkling Stress*	Number of Lobes	Theoretical Maximum Number of Lobes
			End to End in.	Inset Section in.		Center Section	Outer Section	Center Section lb. per sq. in.	Outer Section lb. per sq. in.			
9	B	3.709	8.72	....	.....	0.0251	0.0251	39 100	39 100	0.0914	0	5
12		3.686	8.72	....	.....	0.0213	0.0213	37 700	37 700	0.1036	4	6
10		3.695	8.75	4.75	0.0388	0.0237	0.0241	33 500	31 500	0.0766	3	5
13		3.675	8.72	4.76	0.0389	0.0233	0.0239	34 650	32 300	0.0792	4	5

*Based upon values at outer section where failure occurred.

TABLE 4

RESULTS OF TESTS, FABRICATED SPECIMENS

Specimen No.	Series	Nominal Diameter in.	Length in.	$\frac{t}{R}$ at Ends	Maximum Load lb.	Maximum Average Stress at Ends lb. per sq. in.	Ratio to Theoretical Wrinkling Stress	Number of Lobes	Theoretical Maximum Number of Lobes
816R	E	80	72	0.00598	1 471 000	24 230	0.231	8	11
817R		80	72	0.00581	1 396 000	23 630	0.231	8	11
114		80	72	0.00605	1 607 000	26 180	0.246	8	11
86		80	72	0.00598	1 667 000	27 450	0.261	8	11
710	7	10	30	0.00602	9 850	10 400	0.095	8	12
711		10	30	0.00595	13 880	14 800	0.137	8 to 9	12
721		20	30	0.00304	17 300	9 060	0.164	11 to 12	16
730		30	30	0.00193	24 500	8 960	0.256	10 to 12	20
731		30	30	0.00222	32 000	10 160	0.252	10 to 12	19
740		40	30	0.00150	21 300	5 710	0.210	10	23
741		40	30	0.00153	22 000	5 740	0.207	10	23
750		50	30	0.00121	21 200	4 430	0.202	13	26
751		50	30	0.00118	19 900	4 280	0.200	10 to 13	26
760		60	30	0.00103	24 300	4 160	0.223	10	28
761		60	30	0.00101	22 100	3 870	0.211	10	28
810	8	34	35*	0.00720	265 000	20 000	0.153	irreg.	10
811		34	35	0.00662	270 000	22 200	0.185	7	11
820		34	20	0.00705	299 000	23 200	0.181	6 to 7	10
821		34	20	0.00703	299 000	23 300	0.183	7	10
830		34	6	0.00666	329 000	26 900	0.222	8	11
831		34	6	0.00688	338 000	27 500	0.226	7	11

*Beginning with this specimen the length is expressed in feet.

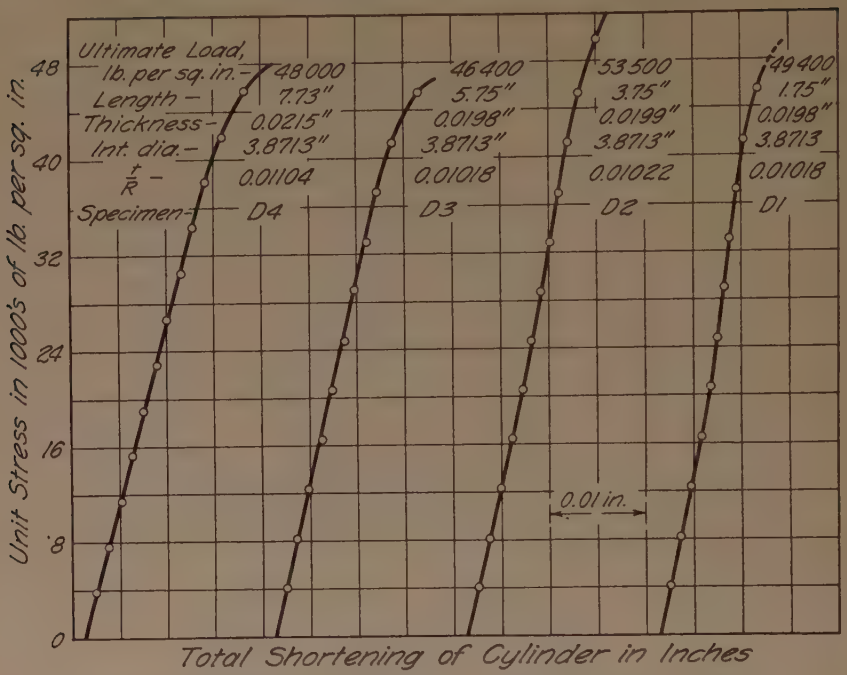


Fig. 3. Stress–Strain Diagrams, 4–in. Shelby Seamless Tubing

effect of the lateral bulging causing a longitudinal shortening of the specimen. This set varies with the thickness of the specimen, being practically negligible for the thin specimens. This is due to the fact that, for the thin specimens, the wrinkles practically completely disappeared upon removal of the load. Specimen 614, the thinnest specimen tested, showed a rather peculiar tendency in that, for loads near the maximum, wrinkles appeared here and there sporadically, each accompanied by a slight drop in load. This is reflected in the rather uneven stress–strain curve. However, after reaching its maximum value, the load dropped off considerably, and wrinkles extended all the way around the specimen. Release of load was accompanied by complete disappearance of the wrinkles. On the second application of load, the same curve was retraced, and the second maximum was almost the same as the first. This, of course, is a characteristic of true elastic wrinkling. Specimen 614 was the only specimen to behave in this manner. For the other specimens the second maximum load reached was usually the same as the load on the specimen immediately after the initial wrinkling.

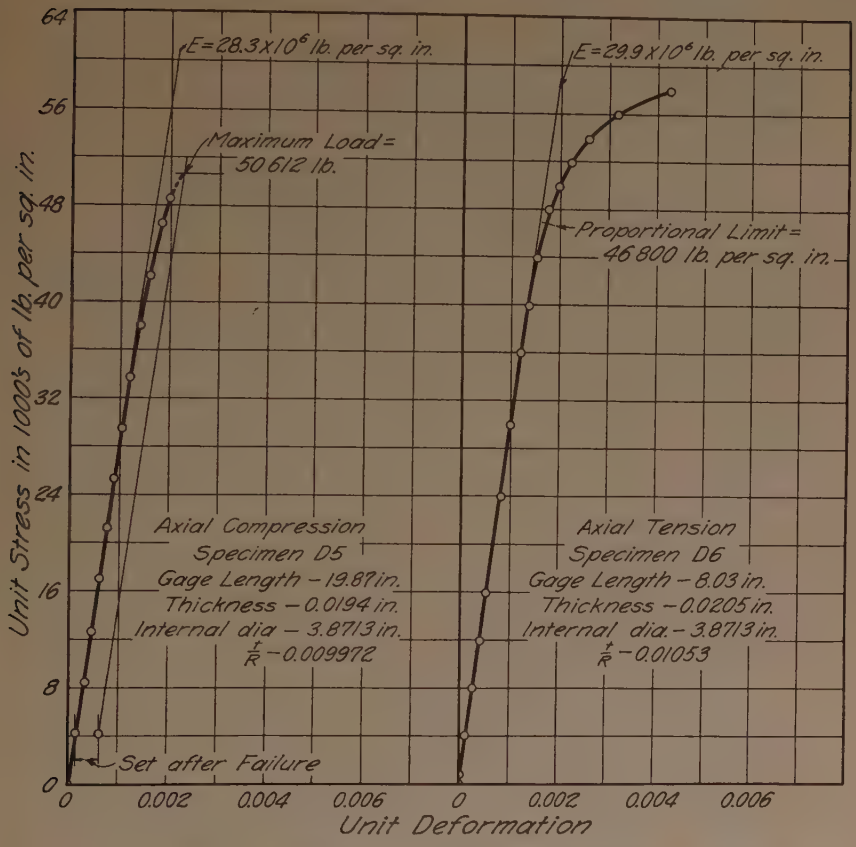


FIG. 4. COMPARISON OF STRESS–STRAIN DIAGRAMS FOR TENSION AND COMPRESSION, 4–IN. SHELBY SEAMLESS TUBING

The longitudinal wave length of deformation, whenever this was clearly defined, was always much less than the radius of the cylinder. Series 6 was the only series for which fairly definite values of this wave length could be obtained. For the few specimens that failed in several rows of wrinkles the distance between adjacent rows, or the wave length, ranged from about 1.50 in. to 2.75 in., and was generally about 2 in. It should be noted here that adjacent rows of wrinkles are always so located that the vertices of the lobes of the upper row are above the centers of the lobes of the lower row. That is, the lobes are staggered in the longitudinal direction.

The physical properties of the material in tension for each series of machined specimens are given in Table 5. The values for each of

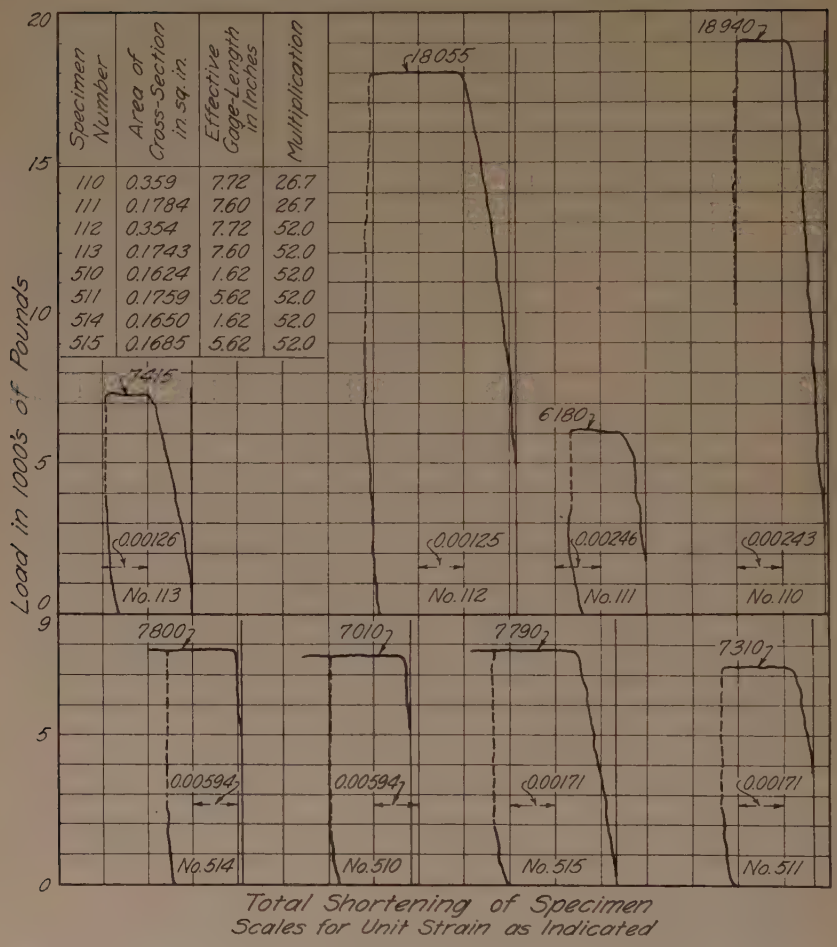


Fig. 5. Autographic Stress–Strain Diagrams, 4–in. Shelby Seamless Tubing

the fabricated specimens are given in Table 6. The stress–strain curve for a control specimen cut from the 14-in. hot-rolled seamless tubing is given in Fig. 7.

12. *Effect of t/R Ratio on Wrinkling Stress.*—In general, for all the different tubes tested, the wrinkling stress seemed to vary practically directly with the t/R ratio up to a certain critical value. This critical value was different for specimens made of different materials. For values of the t/R ratio higher than this critical value the wrinkling stress seemed to be practically constant.

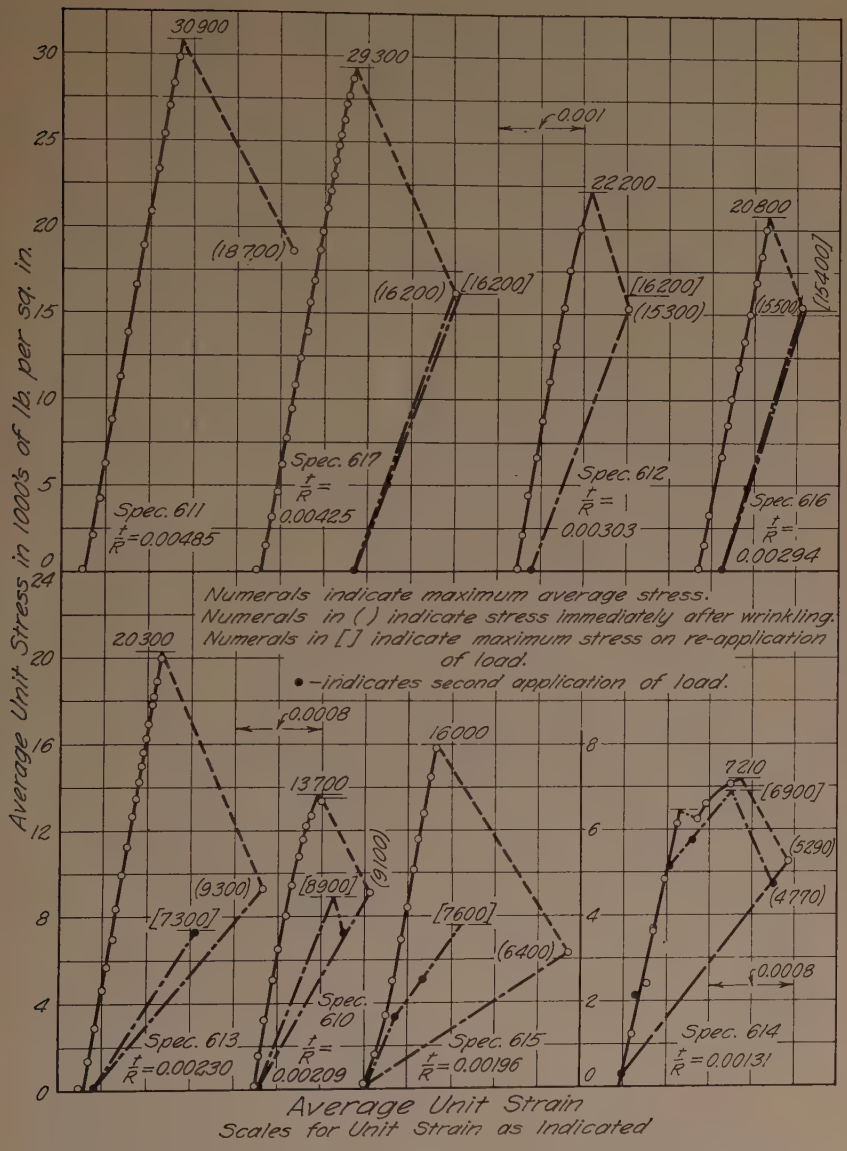


FIG. 6. STRESS–STRAIN DIAGRAMS, 14–IN. HOT–ROLLED SEAMLESS TUBING

TABLE 5

PHYSICAL PROPERTIES OF MATERIAL IN TENSION, MACHINED SPECIMENS

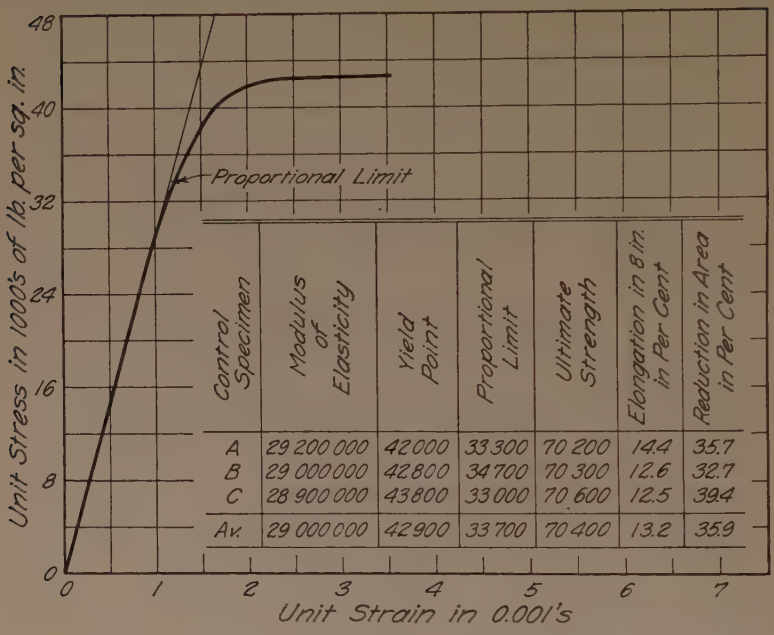
Series	Number of Tests	Type of Control Specimen	Modulus of Elasticity lb. per sq. in.	Proportional Limit lb. per sq. in.	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Elongation per cent	Reduction in Area per cent
A	5	coupons	29 500 000	.....	35 950	58 660	25.3 in 6 in.	....
B	3	coupons	28 200 000	.....	30 500	45 200	8.5 in 6 in.	....
D	1	tube	29 900 000	46 800	.....	.....	.....	....
1	1	tube	29 900 000	46 800	.....	.....	.....	....
2	1	tube	29 400 000	60 800	.....	.....	.....	....
6	3	coupons	29 000 000	33 700	42 900	70 400	13.2 in 8 in.	35.9

TABLE 6

PHYSICAL PROPERTIES OF MATERIAL IN TENSION, FABRICATED SPECIMENS

Series	Specimen No.	Number of Tests Averaged	Modulus of Elasticity lb. per sq. in.	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Elongation in 8 inches per cent	Reduction in Area per cent
E	114	4	29 700 000	37 823	54 600	26.9	51.5
	86	4	29 000 000	36 090	52 120	30.9	53.6
	817R	4	28 600 000	36 085	54 800	27.0	49.4
	816R	4	29 100 000	35 565	56 330	28.2	49.8
7	710	2	28 700 000	25 550	44 300	15.6	....
	711	2	30 100 000*	27 050	43 350	21.3	....
	721	2	30 100 000*	27 050	43 350	21.3	....
	730	2	.....	29 650	48 200	15.0	....
	731	2	.....	32 350	43 750	20.9	....
	740	4	29 900 000†	24 700	45 850	17.6	....
	741	4	29 900 000†	24 700	45 850	17.6	....
	750	4	.....	28 850	48 075	20.2	....
	751	4	30 000 000‡	25 475	46 100	20.6	....
	760	4	.....	31 670†	49 925	16.4	....
	761	4	.....	45 750‡	47 650	11.3	....
8	810	2	29 400 000*	32 800	51 300	23.1	42.3
	811	2	28 200 000	32 400	51 950	25.0	50.8
	820	2	28 700 000	45 150	55 350	18.7	46.7
	821	2	29 800 000	38 100	53 600	27.2	50.0
	830	2	30 100 000	39 950	55 000	26.5	51.3
	831	2	29 100 000	41 000	56 650	16.6	37.9

*One test.
†Average of 2 tests.
‡Average of 3 tests.



Control Specimen	Modulus of Elasticity	Yield Point	Proportional Limit	Ultimate Strength	Elongation in 8 in. in Per Cent	Reduction in Area in Per Cent
A	29 200 000	42000	33300	70 200	14.4	35.7
B	29 000000	42800	34700	70 300	12.6	32.7
C	28 900 000	43800	33000	70 600	12.5	39.4
Av.	29 000000	42900	33700	70400	13.2	35.9

FIG. 7. STRESS–STRAIN DIAGRAMS FOR CONTROL SPECIMENS, 14–IN. HOT–ROLLED SEAMLESS TUBING

The tubes of series A made from lap-welded steel pipe had values of the t/R ratio varying from 0.0122 to 0.0353. The maximum stress for these specimens did not vary greatly from a general average of about 38 500 lb. per sq. in. The thinnest tube of this series failed at a stress of 31 400 lb. per sq. in., but the next thicker tube, having a t/R ratio of 0.0126, sustained a stress of 40 600 lb. per sq. in., the second highest value reached in the series. Probably the critical t/R ratio is somewhat near 0.012 for these tubes.

The specimens of series D and those of series 1 were all machined from the same piece of Shelby tubing, and a comparison may profitably be made, although the end conditions and lengths for the two series of tests were different. The general trend of the results is shown in Fig. 8. It is evident that the thinner tubes failed at stresses less than the thicker ones nearly in proportion to the t/R ratio up to a value of approximately 0.01. The thicker tubes failed at approximately the same stress. It should be noted that the specimens of series D had plain ends and those of series 1 had collars at the ends. This difference in end condition evidently has little if any effect on the wrinkling strength.

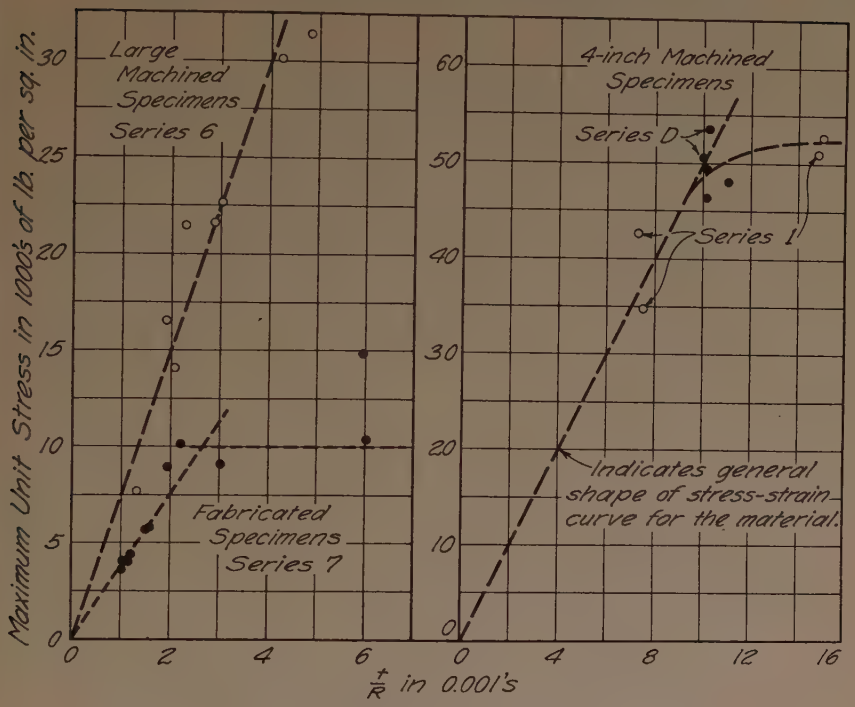

Maximum Unit Stress in 1000's of lb. per sq. in.
Large Machined Specimens Series 6
Fabricated Specimens Series 7
4-inch Machined Specimens
Series D
Series I
Indicates general shape of stress-strain curve for the material.
t/R in 0.001's

FIG. 8. RELATION BETWEEN WRINKLING STRESS AND t/R RATIO

Fig. 8 shows also the t/R ratio–wrinkling-strength relation for the large machined specimens of series 6 and for the fabricated specimens of series 7. The critical t/R ratio evidently has not quite been reached in series 6, the maximum stress varying almost directly as the t/R ratio for all the specimens of the series. For the specimens of series 7, the critical t/R ratio appears to be about 0.0025.

In all cases where the wrinkling stress is approximately proportional to t/R, the constant of proportionality is always much less than that given by Equation (1) for the theoretical critical wrinkling stress. Values of from 30 to 60 per cent of the theoretical are obtained for the machined specimens, and but 20 to 25 per cent of the theoretical values for the fabricated specimens.

For the specimens having a thickness–radius ratio greater than the critical value, the wrinkling strength was practically constant. The stress at collapse, for the machined specimens, was generally above the proportional limit of the material, and, for the lap-welded and wrought iron tubes, at or above the yield point of the material. For the fabricated specimens the maximum stress was considerably

less than the yield point of the tension control specimens for the material. For the very thin fabricated specimens it is felt that this is due partly to uneven distribution of load at the ends of the specimen and to geometrical irregularities in the form of the specimen.

Apparently the only variable having any effect on the number of lobes in the wrinkle causing failure of the specimen is the thickness–radius ratio. Specimens of different length but of the same diameter and thickness had the same number of lobes, and specimens of the same dimensions under different degrees of eccentric loading failed in such a manner as to indicate the probability of there being the same number of lobes in the wrinkle if it had extended completely around the circumference.

In no case was the theoretical maximum number of lobes given by Equation (7) exceeded. However, in all but one of the specimens of 4-in. diameter lap-welded steel tubes, wrought iron tubes, and Shelby seamless steel tubes, of all different thicknesses and lengths, except when the number of lobes was zero, the actual number of lobes was 2 or 3 less than the theoretical maximum. There was no definite correlation between the actual and theoretical number of lobes for the large machined specimens or for the thinnest fabricated specimens.

13. *Use of Empirical Constant in Critical–Strain Method.*—It is evident from the tests reported that the theoretical critical wrinkling stress given by Equation (1) is not at all in accordance with experimental results even in the cases to which it should strictly apply. Since, for elastic wrinkling, Equation (1) and Equation (10) give the same results, neither does the latter equation agree with experiments. However, Equation (10) can be modified by introducing a simple empirical constant so as to give results in fair agreement with those obtained by experiment.

Modifying Equation (10) by the introduction of an empirical constant, c, the critical wrinkling strain is given by the relation

$$\epsilon = c \times 0.60 \frac{t}{R} \tag{12}$$

Using this relation, the scale may be established whereby the stress–strain curve for the material in compression becomes also the wrinkling-strength–t/R curve.

Experimentally determined values of c for the Shelby tubing of series 1 and series D are given in Table 7. The unit strain at maximum load obtained from the stress–strain curves for the tubes is unreliable, but it is given to show that there is some agreement be-

TABLE 7

WRINKLING STRESS FOR SHELBY TUBING OF VARIOUS THICKNESSES: COMPARISON OF EXPERIMENTAL AND COMPUTED VALUES

Specimen No.	$\frac{t}{R}$ Ratio	Measured Unit Strain at Maximum Load	Corrected Maximum Stress* lb. per sq. in.	Strain Corresponding to Maximum Stress†	Theoretical Critical Strain	Ratio of Maximum Strain to Theoretical Strain	Ratio of Maximum Stress to Theoretical Stress
110	0.01503	0.0022	55 500	0.00301	0.00910	0.331	0.204
111	0.00752	0.0022	40 400	0.00137	0.00455	0.302	0.297
112	0.01482	0.0021	52 600	0.00235	0.00896	0.262	0.196
113	0.00736	0.0016	43 700	0.00152	0.00445	0.342	0.328
D1	0.01018	.....	49 400	0.00192	0.00616	0.312	0.267
D5	0.00997	0.0023	50 600	0.00205	0.00604	0.340	0.280
Average.....	.....	.....	.....	.....	.....	0.315	.....

*From Table 2.
†From stress-strain diagram from control specimen.

tween this value and the value obtained from the control specimens. Strictly, the stress–strain curve for the material in compression should be used, but this is generally not convenient to obtain. It is evident from the table that, although there is a large variation in the ratio of actual to theoretical stress, there is only a small variation in the ratio between actual and theoretical strains. The average value of c for this tubing, from Table 7, is 0.315. This is based upon the strain corresponding to the maximum unit stress corrected for variations in the thickness of the shell. This value should be reduced to allow for imperfections, if they are to be expected. A value of 0.27 would give results that correspond more closely to the actual wrinkling stresses uncorrected for imperfections for the more irregular tubes of this series.

To obtain the wrinkling-stress–t/R relation from the stress–strain curve for, say, a value of $c = 0.30$, the t/R scale would be $\frac{1}{0.18}$ times the strain scale (see Equation (11)). For this value of c the following results were obtained:

Specimen No.	t/R ratio	Predicted wrinkling stress, $c = 0.30$	Actual maximum stress
110	0.01503	54 500	55 500
111	0.00752	40 400	40 400
112	0.01482	54 300	52 600
113	0.00736	40 000	43 700
D1	0.01018	48 800	49 400
D5	0.00997	48 400	50 600

The value of c for the lap-welded steel pipe of series A is indeterminate, since failure occurred at or near the yield point where there is a large variation in strain with little variation in stress. Similarly, the specimens of series 2 failed at a stress corresponding to a point beyond the rather sharp break in the stress–strain curve for the control specimen.

For all the other specimens the actual stress at collapse was less than the proportional limit of the material, although, except for one specimen of series 6, failure took on the characteristics of plastic failure in that recovery was not complete when the load was removed.

For specimens for which failure is *elastic* the critical-stress method, represented by Equation (1), and the critical-strain method, represented by Equation (11), are equally applicable and give the same results. For material not having a well-defined yield point and for values of t/R that cause failure to occur at a stress somewhat above

the proportional limit, the critical-strain method appears to be particularly useful.

The experimentally determined values of c for the various groups of specimens are:

Series	Type of Specimen	c
1 and D	4-in. Shelby tubing	0.31
5	4-in. Shelby tubing	0.37
6	14-in. hot-rolled tubing	0.45
7	Fabricated specimens thickness = 0.03 in.	0.20 to 0.25
8	Fabricated specimens thickness = $\frac{1}{8}$ in.	0.18 to 0.23
E	Large fabricated specimens thickness = $\frac{1}{4}$ in.	0.25 to 0.26

14. *Effect of Length on Wrinkling Stress.*—Theoretically, length has no effect on the critical wrinkling stress of a tube. However, for a long tube, flexure of the axis will occur and Euler buckling, or a combination of buckling with wrinkling, will take place. The length to which pure wrinkling failure is limited is not definitely known. There are several difficulties attached to the experimental determination of this length. Imperfections in the tube, irregularities in thickness, lack of homogeneity of the material, etc., are more likely to occur in a long tube than in a short one.

Machined and fabricated specimens were tested to observe whether, over a short range, the length has any effect on the wrinkling stress. The machined specimens of series D were fairly regular in thickness and showed little variation in strength for a variation in length from 1.75 to 20 inches. The specimens of series 5 were irregular, and since they were thinner than those of series D, the irregularities were of greater importance. Nevertheless, the longest specimen of this series attained the highest stress and was less than the average in thickness. The maximum stresses corrected for irregularities in thickness are more nearly uniform. The relation between the length and the wrinkling stress for these two series of specimens are given in Fig. 9. The horizontal lines indicate the average value of the wrinkling stress for each series.

The fabricated specimens of series 8 gave a decrease in strength with length. This is shown in Fig. 10. It is a question how much of this may be ascribed to flaws, dents, variations in thickness, and other irregularities in the longer specimens. The initial bend of about $\frac{3}{4}$ inch in the axis of one of the longest specimens would increase the

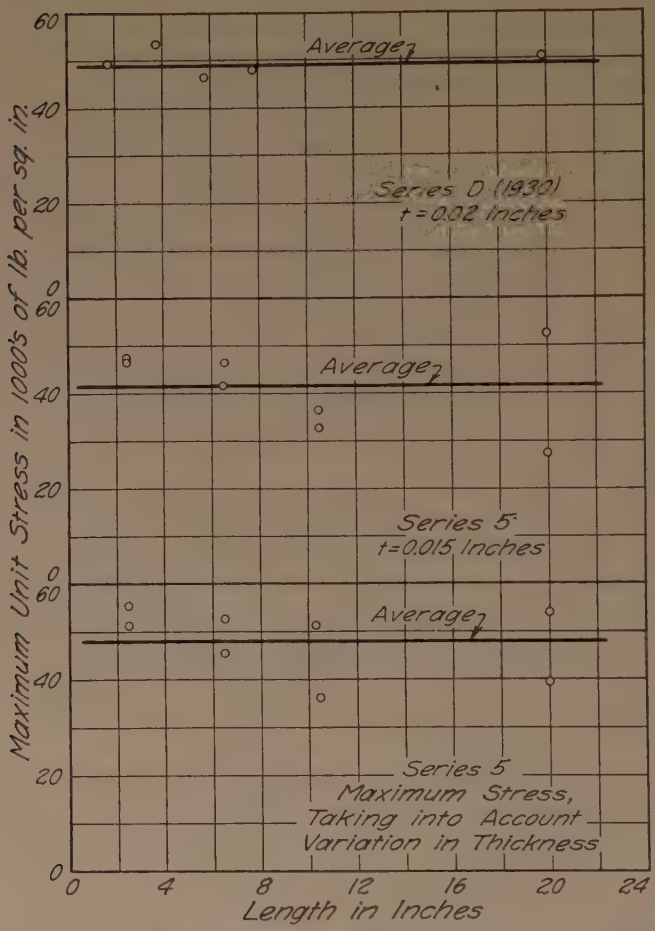


Fig. 9. Relation Between Length and Wrinkling Stress, 4–in. Shelby Seamless Tubing

stress on one side by practically 9 per cent. The difference between the load carried by the 6-ft. specimens and that carried by the 35-ft. specimens is about 20 per cent. Variation in thickness, the weakness of transverse joints, dents, and irregularities may have caused part of this difference. The material in the 35-ft. specimens at the section of failure had a lower yield point than the material in the shorter specimens. This may have caused failure to occur at a low load even though the average wrinkling stress was below the yield point.

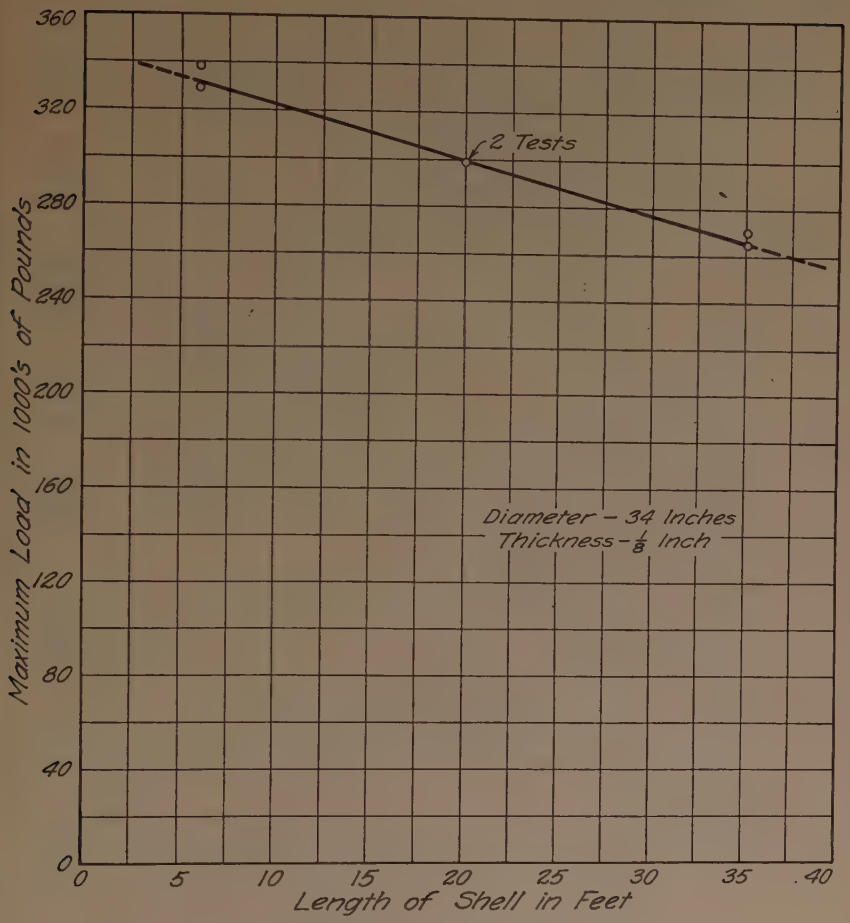


FIG. 10. RELATION BETWEEN LENGTH AND WRINKLING STRESS, FABRICATED SPECIMENS, SERIES 8

It should be noted in this connection that all the long specimens failed at a transverse joint or at a dent in the shell wall. The 6-ft. specimens failed at the ends of the tube. It is believed that the decrease in load carrying capacity for the longer specimens is attributable to unavoidable physical and geometrical imperfections in long fabricated cylinders rather than to the direct effect of length.

15. *Effect of Eccentric Loading on Wrinkling Stress.*—If the deformations in the shell wall which produce a condition of instability resulting in wrinkling are a direct linear function of the longitudinal

TABLE 8

WRINKLING STRESS FOR ECCENTRICALLY-LOADED SHELLS, SERIES 2

Specimen No.	Internal Diameter in.	Length		$\frac{t}{R}$	Eccentricity of Load in.	Maximum Average Stress lb. per sq. in.	$\frac{P}{A}\left(1+\frac{2e}{R}\right)$ lb. per sq. in.	Corrected Stress lb. per sq. in.	Least Thickness On Side Where Initial Failure Occurred
		End to End in.	Between Shoulders in.						
210	3.874	4.50	3.48	0.01018	0.01	63 400	64 000	65 200	0.0194
211	.....	4.49	3.50	0.01001	0.25	52 600	66 100	64 200	0.0195
212	.....	4.49	3.52	0.00997	0.50	45 200	68 500	68 900	0.0194
213	.....	4.49	3.50	0.00996	0.75	39 500	70 000	66 500	0.0194
214	.....	4.48	3.50	0.00952	1.00	33 200	67 500	70 400	0.0167
215	.....	4.50	3.50	0.00956	0.26	50 500	64 000	69 200	0.0162
216	.....	4.47	3.52	0.00981	0.01	57 200	57 700	61 000	0.0167
217	.....	4.49	3.50	0.00936	0.50	46 800	71 000	69 000	0.0182
218	.....	4.50	3.50	0.00977	0.75	40 650	72 100	66 500	0.0190
219	.....	4.47	3.52	0.00952	1.00	33 080	67 300	67 300	0.0185
Average	.....	.....	.....	0.00977	....	.....	66 800	66 800	0.0185
220	3.874	16.00	15.02	0.00972	1.00	28 050	57 000	64 100	0.0149
221	.....	16.01	15.02	0.00966	0.50	36 950	56 000	60 000	0.0162
223	.....	15.97	15.05	0.00946	0.50	42 200	64 000	66 000	0.0176
224	.....	15.97	15.01	0.01022	0.01	57 500	58 100	61 000	0.0182
226	.....	16.01	15.00	0.00941	0.01	40 030	40 400	55 700	0.0118
227	.....	15.98	15.03	0.01032	0.01	48 700	49 200	55 100	0.0160
228	.....	15.98	15.03	0.01002	1.00	24 700	50 200	52 200	0.0190
229	.....	15.98	15.03	0.01008	0.01	52 200	52 800	55 700	0.0181
Average	.....	.....	.....	0.00986	....	.....	53 500	58 700	0.0165

strains in the shell, the cylinder will collapse by wrinkling when the strain at any point reaches the critical strain. Assuming a linear relation between stress and strain, the maximum stress for a given load and a given eccentricity is determined by the relation

$$f' = \frac{P}{A}\left(1 + \frac{2e}{R}\right) \tag{13}$$

where f' = maximum stress
P = load on the cylinder
A = area of cross-section of shell wall
e = eccentricity of load
R = mean radius of shell wall

Tubes of two different lengths, but of the same diameter and thickness were tested under different degrees of eccentric loading. Measurements were taken of the longitudinal deformations on four gage lines around the circumference, one gage line being in the direction of eccentricity of load. There was an approximately linear distribution of strain across the section of the cylinder.

The wrinkling stresses for the eccentrically loaded shells are given in Table 8. The correction for variation in thickness was in some cases negative for the 4-in. cylinders. For the longer cylinders it was always positive since the cylinders were so irregular that there was always at least one point where the effect of the two eccentricities was additive.

The results of the tests are plotted in Fig. 11. The average stress at collapse, the maximum stress computed by Equation (13) from the average stress at wrinkling, and this maximum corrected for variations in thickness of the shell wall, are all shown plotted against the eccentricity of load. The average-stress–eccentricity relation is practically a linear one. The maximum stresses computed by Equation (13) are slightly erratic, but the corrected stresses seem to be approximately uniform. The difference in strength of the two different lengths of specimens is not consistent with the results shown in Fig. 9. This difference in strength may be ascribed to imperfections in the tubes, to the manner of putting the longer tubes on the mandrel,* or to an inherent weakness of long specimens as compared with short ones.

However, the conclusion is apparently justified that for tubes similar in all respects except for degree of eccentricity of load, wrin-

*The long tubes were heated slightly with a blow torch in putting them on the mandrel. It is believed that they were not sufficiently heated to affect their physical properties.

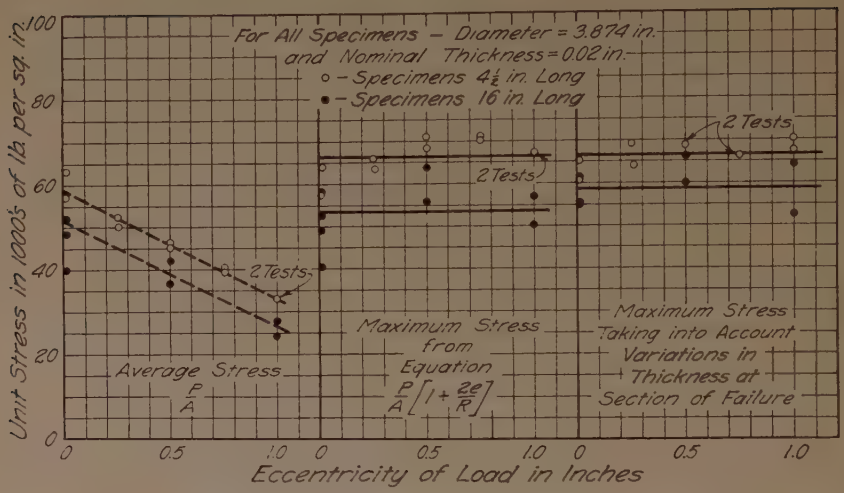

For All Specimens – Diameter = 3.874 in. and Nominal Thickness = 0.02 in.
o – Specimens 4½ in. Long
• – Specimens 16 in. Long
2 Tests
2 Tests
2 Tests
Average Stress P/A
Maximum Stress from Equation P/A [1 + 2e/R]
Maximum Stress Taking into Account Variations in Thickness at Section of Failure
Unit Stress in 1000's of lb. per sq. in.
Eccentricity of Load in Inches
0
20
40
60
80
100
0.5
1.0

Fig. 11. Relation Between Wrinkling Stress and Eccentricity

kling will take place at a certain maximum stress, f', as computed by Equation (13), for all degrees of eccentricity. In other words, eccentricity of load has apparently no effect beyond that of changing the distribution of stress on the cross-section of the cylinder.

16. *Wrinkling of Fabricated Cylinders.*—It is to be expected that cylindrical shells fabricated in the ordinary manner will be weaker than machined shells. Irregularities in material affect both types of specimens; but fabricated specimens are, in addition, affected by transverse joints, dents, flaws, and geometrical imperfections such as bends in the axis and irregularity in the shape of cross-section.

For the large fabricated specimens of series E, the unevenness in bearing was reflected in the irregular distribution of strain around the circumference of the cylinder as shown by the strain measurements. Filling the gaps with shims, as in series 8, gave a better distribution of strain, but even in this series the maximum strains were in general about 14 per cent greater than the average for loads near the wrinkling load.

The change in profile during the test was very slight except for the cylinders having a lap-riveted girth seam. It should be noted that the effect of the lap joint is evident at fairly low loads. Transverse lap joints are a serious source of weakness; the stress sustained by the lap-riveted cylinders of series E was 10 to 12 per cent lower than that carried by the corresponding butt-welded cylinders. The

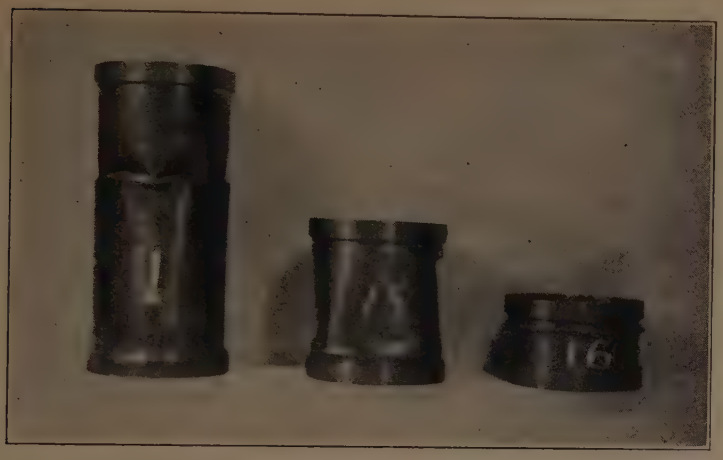


FIG. 12. LAP-WELDED STEEL TUBING AFTER FAILURE

small machined wrought iron cylinders with offset walls were weaker than the corresponding straight cylinders by about 15 to 20 per cent, indicating that the difference in strength is due not to riveting of the joint, but to the offset in the shell wall.

It is not definitely known how much butt-welded joints affect the strength of a cylinder. However, all the fabricated specimens having transverse joints failed initially at a joint except in the case of one specimen which failed at an evident flaw or dent in the shell.

All the fabricated specimens from short flimsy ones to long heavy ones failed at practically the same proportion of the theoretical load, and at a stress considerably lower than the yield point of the material.

17. *Character of Failure.*—The failure of the machined specimens, the ones most nearly approaching geometrical perfection, was abrupt and without warning. The geometrical imperfections and localized stresses caused impending failure of the fabricated specimens to be more apparent. Even for the latter, however, final failure was very abrupt.

The symmetry of the lobed specimens after failure was a continuous source of wonder. Some of the more perfect geometrical figures are shown in Figs. 12 to 18, inclusive.

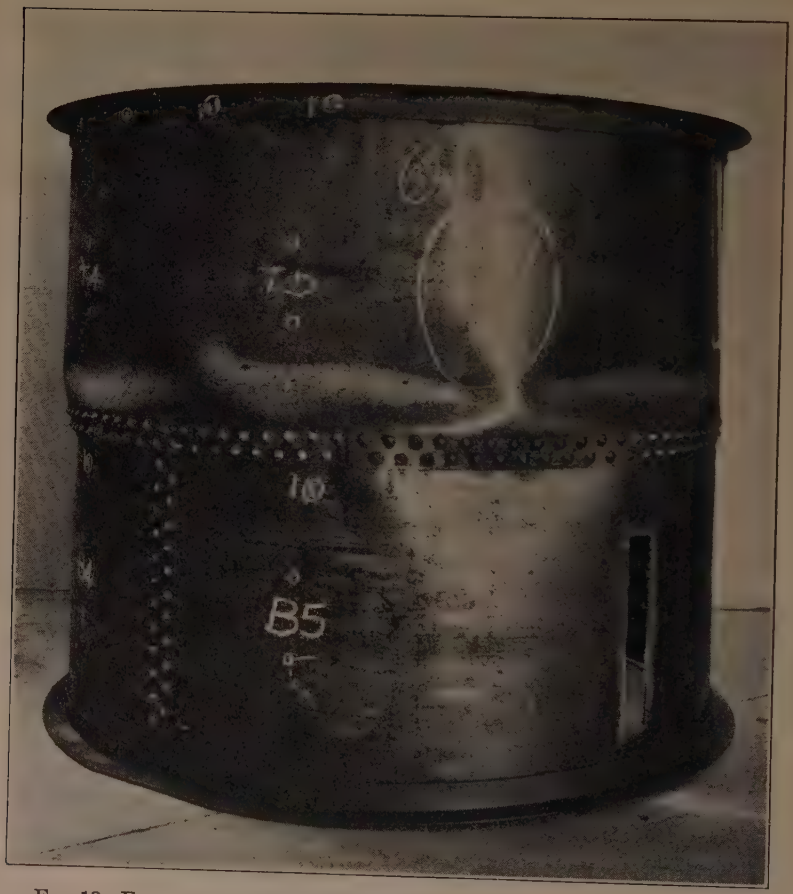


FIG. 13. FABRICATED SPECIMEN 80 INCHES IN DIAMETER AND 0.25 INCH IN THICKNESS AFTER FAILURE

FIG. 14. FABRICATED SPECIMEN 35 FEET IN LENGTH, 34 INCHES IN DIAMETER, AND 0.125 INCH IN THICKNESS IN TESTING MACHINE; WRINKLING FAILURE OCCURRED AT MIDDLE OF LENGTH

FIG. 15. MACHINED SPECIMENS 14 INCHES IN DIAMETER AFTER FAILURE

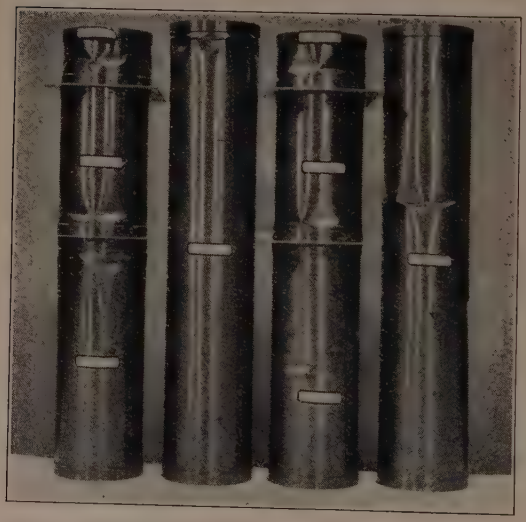


FIG. 16. 4-IN. SHELBY SEAMLESS TUBING AFTER FAILURE

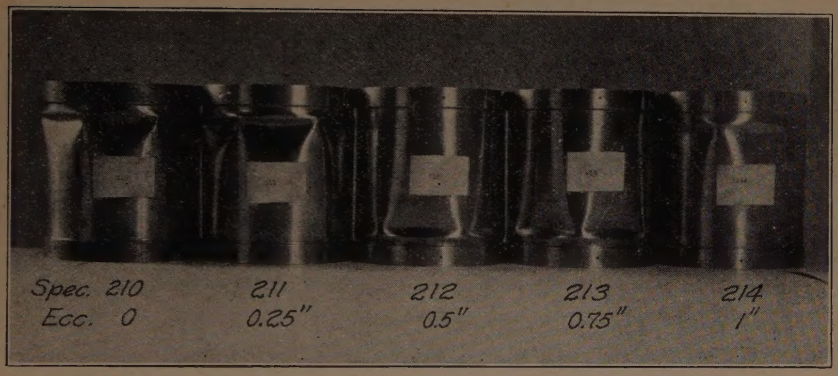


FIG. 17. FAILURE UNDER DIFFERENT DEGREES OF ECCENTRIC LOADING; 4-IN. SHELBY SEAMLESS TUBING SPECIMENS 4 INCHES IN LENGTH

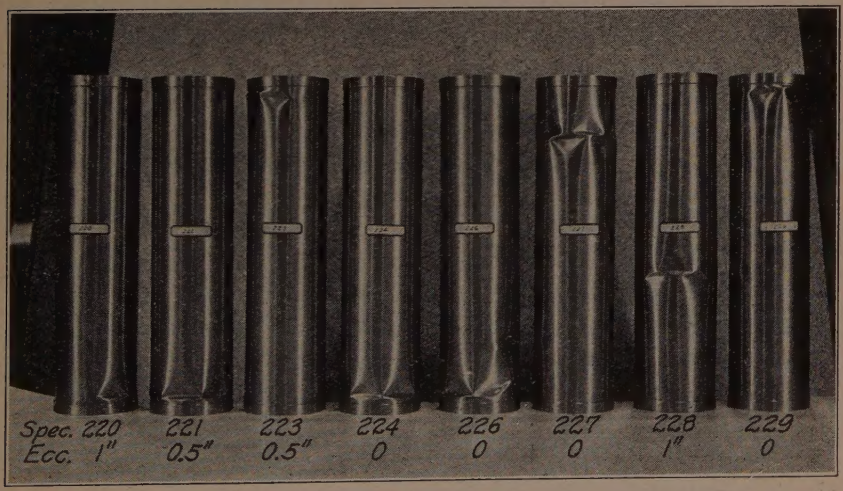


FIG. 18. FAILURE UNDER DIFFERENT DEGREES OF ECCENTRIC LOADING; 4-IN. SHELBY SEAMLESS TUBING SPECIMENS 16 INCHES IN LENGTH

VI. Summary

18. *Summary.*—The following conclusions appear to be justified by the tests reported herein:

(1) The theoretical wrinkling stress was not reached in any of the tests. The highest value obtained was about 60 per cent of the theoretical value for one of the most carefully machined specimens. In general, a series of incipient wrinkles may be present before failure, and any one of these, probably the one at the weakest point of the shell, will cause collapse. It is not to be expected that the values given by Equation (1) can ever be reached. This large discrepancy between the theoretical and experimental values of the wrinkling stress was also obtained by previous investigators.

(2) The wrinkling strength of the cylinders tested varied approximately directly with the t/R ratio up to a certain critical value of this ratio. This critical value was at least 0.005 for the large machined specimens, about 0.01 for the small machined specimens, and about 0.003 for the thin fabricated specimens. For higher values of the t/R ratio the wrinkling strength remained practically constant.

(3) The ratio of the strain corresponding to the wrinkling stress to the theoretical wrinkling strain given by the equation $\epsilon = 0.60\ t/R$ was about 0.31 to 0.37 for small specimens machined from Shelby tubing, about 0.45 for large carefully machined specimens, and ranged from 0.18 to 0.26 for fabricated specimens.

(4) For 4-in. diameter machined specimens, varying the length from 1¾ in. to 20 in. has no effect upon the wrinkling stress providing the specimens are uniform in thickness.

(5) There is a decrease in strength of fabricated specimens with length. This decrease is believed to be due to variations in thickness, flaws, and irregularities, which also increased with length. The difference in strength between 6-ft. and 35-ft. specimens 34 in. in diameter and ⅛ in. thick was about 20 per cent.

(6) The maximum stress at collapse for cylindrical shells eccentrically loaded, computed by the formula $f = \frac{P}{A}\left(1 + \frac{2e}{R}\right)$, is practically constant for different degrees of eccentric loading within the range in $\frac{e}{R}$ from 0 to ½.

(7) The number of lobes was in all cases less than the theoretical maximum number given by Equation (7). In general, except for specimens having very small values of t/R, the actual number was

from 2 to 4 lobes less than the theoretical maximum, for both machined and fabricated specimens.

(8) A transverse lap joint in a cylindrical shell is a source of weakness causing failure at a load from 10 to 20 per cent lower than for a corresponding straight shell. Moreover, the bulging or wrinkling at the joint starts at low loads.

(9) Nearly all fabricated specimens failed either at the ends, at transverse joints, or at flaws or dents in the shell.

(10) Failure was in almost all cases sudden and accompanied by large vertical deformation causing a sudden drop in load on the testing machine. For fairly thin specimens having no great defects there is little if any warning of impending failure.

(11) The following rules are suggested for the working stress, P, to be used in designing thin cylindrical steel shells that function as columns for which the ratio of the length to the radius of gyration does not exceed 35:

For machined specimens or cold drawn tubing, straight and free from local indentations,

$P = 2\,000\,000 \frac{t}{R}$ lb. per sq. in., but not to exceed one-half of the proportional limit of the material in tension.

For fabricated specimens, straight and free from local indentations, and containing no lap girth seams,

$P = 1\,600\,000 \frac{t}{R}$ lb. per sq. in., but not to exceed one-third of the yield point of the material in tension.

The working stress, P, is the maximum stress to be found on any portion of any section. It includes the average stress over the transverse section due to an axial load, the flexural stress due to eccentricity of the axial force or other causes, and any localized stress due to the manner of loading or supporting the cylinder.